A Novel Curve-Fitting Algorithm Based on Levenberg-Marquardt Method of Single-Diode Model Photovoltaic Modules

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Abstract

Aiming at matrix singularity and decrease accuracy among the processing of curve-fitting algorithm of single-diode model photovoltaic (PV) modules, this paper proposes a novel curve-fitting algorithm based on Levenberg-Marquardt Method. According to the sample data with the same voltage sampling interval of monocrystalline silicon photovoltaic module and polysilicon photovoltaic modules under standard environment, practical results shows that the proposed curve-fitting algorithm has the advantage of convergence effectively and broad applicability. Besides, comparing with the previous curve-fitting algorithm based on the Modified Newton method and Reduced-forms, proposed algorithm has a superior advantage of high accuracy.

Keywords: *Photovoltaic modules, curve-fitting, Levenberg-Marquardt method.*

I. Introduction

For PV systems designers, it is indispensable to develop suitable models to closely simulate characteristics of PV cells and to simulate, design, evaluate, control and optimize PV systems effectively[1, 2].

Derived from material structure of photovoltaic modules, single-diode model of PV can describe the complex characteristic relationship of photovoltaic modules, which has been widely used for the good balance between model accuracy and algorithm complexity [3, 4].

Based on single-diode model of photovoltaic modules, simulation accuracy of curve-fitting algorithm is highest [5, 6]. However, in the process of extracting model characteristic parameters, curve-fitting algorithm has the problem of inverse matrix singularity (unexpected oscillation and terminal value overflow) [7]. In order to avoid the matrix singularity of curve-fitting algorithm, literature [8] reduces the parameters to be solved from five to three through the functional relationship of special points, and conditionally adjusts the diode ideal factor with additional equations based on the principle of Newton method. Literature [9] reduces the parameters to be solved from five to two through the functional relationship of special points to ensure the effective convergence of the curve fitting algorithm. It is proved that objective function has a unique optimal solution under two characteristic parameters, and two methods to reduce the dimension of algorithm are given in literature [10]. However, reducing the number of parameters undetermined in the optimization process contribute to the decrease of simulation accuracy.

In order to avoid matrix singularity problem and decline of curve fitting accuracy in the algorithm convergence process, this paper selects appropriate parameters to be solved and decouples the implicit transcendental equation of the model by using Lambert W (Lambert) function [11], using the method for non-linear least squares problems to seek optimal error between theoretical value and measured value. Curve-fitting algorithm rely on the least squares error to find an optimized fit between theoretical and experimental I–V characteristics of solar cells. The Levenberg–Marquardt Method same as other parameters extraction techniques (Newton's Method or Gauss–Newton Method) is used to fit the curve of uniformly sampled data to extract the characteristic parameters

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contained in the objective function.

In this paper, to verify the effect of algorithm solution under standard environment (1000W/m2, 25℃), the test data of monocrystalline and polycrystalline silicon module at uniform voltage interval are fitted. And the curve fitting algorithm based on modified Newton method [8] and Reduced-forms curve fitting algorithm were compared [10]. It is proved that the curve-fitting algorithm proposed in this paper has wide application and higher solution accuracy.

II. The Single-diode Model of Photovoltaic Modules

The theoretical characteristic curve of electric energy of current, power and voltage output by photovoltaic modules under standard environment is shown in Fig. 1, and equivalent circuit is shown in Fig. 2.

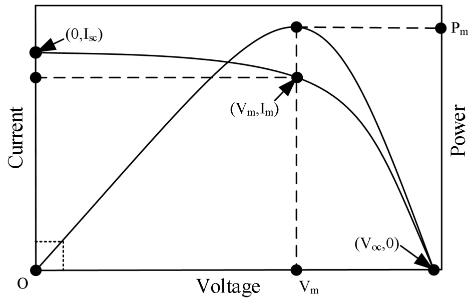

Fig. 1 I-V、P-V curve of photovoltaic module

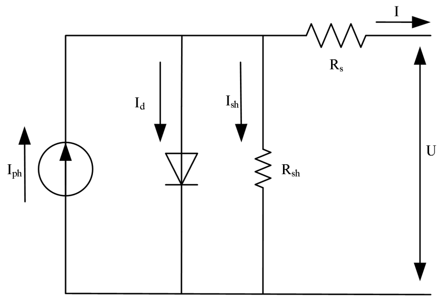

Fig. 2 Single-diode model equivalent circuit diagram of PV module

The mathematical expression of equivalent circuit model I-V of single-diode model is as follows:

$$I = I_{ph} - I_0\left(exp\left(\frac{U + IR_s}{nU_{th}}\right) - 1\right) - \frac{U + IR_s}{R_{sh}} \tag{1}$$

According to Eq.(1), photo-generated current Iph, diode reverse saturation current I0, diode ideal factor n, series resistance Rs and parallel resistance Rsh are the characteristic parameter contained in the single-diode model of photovoltaic module. Uth is thermoelectric pressure Uth=KTNs/q，Ns is the number of cells in the module, K is the Boltzmann constant, T is the operating Kelvin temperature of the photovoltaic module, and q is the amount of electron charge.

To avoid matrix singularity problem and to simplify curve-fitting algorithm complexity, open circuit point（Uoc, 0）, reverse saturation current I0 of the diode with a large difference between the analytical order of magnitude

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and other parameters to be solved. After open circuit point is brought into Eq.(1), diode reverse saturation current I0 can be obtained by Eq.(2):

$$I_0 = \frac{I_{ph} - \frac{U_{oc}}{R_{sh}}}{exp\left(\frac{U_{oc}}{nU_{th}}\right) - 1} \tag{2}$$

Eq.(3) can be obtained by decoupling Eq.(1) using Lambert W function[12]:

$$\begin{aligned} I &= \frac{R_{sh}(I_{ph}+I_0)-U}{R_{sh}+R_s} - \frac{nU_{th}}{R_s} lambertW(x) \\ x &= \frac{I_0 R_{sh} R_s}{nU_{th}(R_{sh}+R_s)} exp\left(\frac{(I_{ph}+I_0)R_{sh}R_s + R_{sh}U}{nU_{th}(R_{sh}+R_s)}\right) \end{aligned} \tag{3}$$

Replacing the diode reverse saturation current I0 in Eq.(3) with Eq.(2), to eliminate the numbers of unknown parameters, equation can be reformed as Eq.(4):

$$I_i^{cal} = \theta_1 - \frac{U_i}{R_{sh}+R_s} - \frac{nU_{th}}{R_s} lambertW(\theta_2) \tag{4}$$

Where,

$$\begin{aligned} \theta_1 &= \frac{I_{ph} R_{sh} exp\left(\frac{U_{oc}}{nU_{th}}\right) - U_{oc}}{\left(exp\left(\frac{U_{oc}}{nU_{th}}\right)-1\right)(R_{sh}+R_s)} \\ \theta_2 &= \frac{I_{ph} R_{sh} R_s - U_{oc} R_s}{nU_{th}\left(exp\left(\frac{U_{oc}}{nU_{th}}\right)-1\right)(R_{sh}+R_s)} \cdot exp(\theta_3) \\ \theta_3 &= \frac{I_{ph} R_{sh} R_s exp\left(\frac{U_{oc}}{nU_{th}}\right) - U_{oc} R_s}{nU_{th}\left(exp\left(\frac{U_{oc}}{nU_{th}}\right)-1\right)(R_{sh}+R_s)} + \frac{R_{sh} U_i}{nU_{th}(R_{sh}+R_s)} \end{aligned}$$

Uoc、Uth is a constant, the superscript cal of Iical represents the theoretical calculated current, and the subscript i refers to the i th group of N groups of data.

III. Parameters Extraction of the Single-diode Model of PV

In this section, the single-diode model, which mainly includes the classical equation by Lambert W function and principle of LM algorithm was introduced. And then the characteristic of LM algorithm and the way of precision improvement were analyzed.

3.1. Single-diode PV Model and Evaluation Parameters

Curve-fitting algorithm relies on least square method to find an optimized fit between theoretical and experimental I-V characteristics of solar cells. The curve-fitting problem of single-diode model can be described as finding the optimal parameters by minimizing square sum of the error between theoretical calculated current and actual measured current. The objective function is defined as follows：

$$S(x_k)=min\left(\sum_{i=1}^{N}(I_i^{cal}-I_i^{exp})^2\right) \tag{5}$$

Considering five parameters of model in Eq.(5), x=[Iph, Rs, R-1 sh, n]. N is the number of sampling points, Ical i、 Iexp i is theoretical calculated current and experimental measured current at the i th sampling point. Define the evaluation parameter RMSEI (Eq. (6)) according to the objective function Eq. (5)

$$\mathrm{RMSEI}=\sqrt{\frac{\sum_{i=1}^{N}(I_i^{cal}-I_i^{exp})^2}{N}} \tag{6}$$

The smaller the numerical evaluation parameter value is, the more accurate parameters extraction result of the curve fitting algorithm is. Therefore, root mean square error RMSEI can predict model accuracy to describe the experimental data.

3.2. Algorithm Flow

According to curve-fitting algorithm of single-diode model photovoltaic module for objective equation Eq. (4), the algorithm steps of the curve fitting algorithm are as follows：

1) Input the measured data and the number of unit cells of a module, define the unknown parameters vector x0=[Iph, Rs, R-1 sh, n];

2) Set the iteration sequence number k=0 , parameter λ=2 , convergence parameter ε=10-6, define Jacobian matrix Jk and initialize parameters μk=10-6·max(JT k· Jk);

$$J_k=\left[\frac{\partial I_k^{cal}}{\partial I_{ph}},\frac{\partial I_k^{cal}}{\partial R_s},\frac{\partial I_k^{cal}}{\partial R_{sh}^{-1}},\frac{\partial I_k^{cal}}{\partial n}\right]$$

3) Solving iteration step dk by the equation as follows:

$$(J_k^T\cdot J_k+\mu_k\cdot \boldsymbol{I})d_k=-J_k^T\cdot F_k$$

Among this, parameter I is unit matrix, and the current error $F_k=I^{cal}-I^{exp}$;

4) If dk less than ε, step to 7， else step to 5；

5) Calculate parameter xnew and ρ by the follows;

$$x_{new}=x_k+d_k$$

$$\rho=\frac{F_k-F_{new}}{0.5\cdot d_k^T(\mu_k d_k-J_k^T\cdot F_k)}$$

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6) Calculate symmetric matrix μ:

If ρ≤0，

$$\mu_{k+1} = \mu_k \cdot \lambda$$
$$\lambda = \lambda \cdot 2$$

If ρ>0，

$$\mu_{k+1} = \mu_k \cdot max\left\{\frac{1}{3}, 1-(2\rho-1)^3\right\}$$
$$x_{k+1} = x_{new}$$
$$\lambda = 2$$

7) Calculate reverse saturation current I0

$$I_0 = \frac{I_{ph} - \frac{U_{oc}}{R_{sh}}}{exp\left(\frac{U_{oc}}{nU_{th}}\right) - 1}$$

Output characteristic parameters $I_{ph}, I_0,\ R_s, R_{sh}, n$.

3.3. Algorithm Analysis

LM method proposed in this paper is different from commonly used Gauss-Newton method in curve-fitting algorithm. It can effectively adjust iteration step size and ensure descent iteration direction, so that algorithm can combine the characteristics of Gauss-Newton method and gradient descent method. Secondly, in order to avoid matrix singularity in solving the inverse matrix due to the small value of diode reverse saturation current and decline of simulation accuracy caused by the excessive reduction of parameters to be solved, parameters number to be solved is reduced from five to four, which ensured algorithm can converge effectively and high simulation accuracy.

IV. Numerical Results

4.1. Case Study Data Information and Evaluation Parameters

In this paper, LM method is verified by uniformly sampling test data of monocrystalline silicon module (JKM280) and polycrystalline silicon module (JKM315) at a voltage interval of about 1V in a standard environment (1000W/m2, 25℃).Data information of case is shown in Table1.

Table 1：Data information of case

	JKM280	JKM315
Rated Power (*w*)	280	315
Number of cells (*pcs*)	60	72
Cell/module material	monocrystalline	polysilicon
Sampling points number(*N*)	40	46

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4.2. Case Study Results

In this paper, initial values of Iph, Rs, Rsh, n are respectively set as short-circuit current, 0.2 ohm, 200 ohm and 1.3. LM method successfully extracts the characteristic parameters of the case study, and which also be compared with the Reduced-form method (RF) proposed in literature [10] and the modified Newton method (NR) proposed in literature [8]. Comparative data results are shown in Table 2.

Table 2 Experiment Results

Module Type	Method	I_{ph} (A)	I_0 (A)	R_s (Ω)	R_{sh} (Ω)	n	RMSEI
JKM280	RF	9.4616	5.19×10^{-13}	0.2847	203.7388	0.7691	31.5498×10^{-3}
	NR	9.4609	5.58×10^{-12}	0.2661	201.3881	0.8340	26.5474×10^{-3}
	LM	9.4592	5.74×10^{-12}	0.2659	204.4556	0.8348	26.5395×10^{-3}
JKM315	RF	8.9765	1.30×10^{-9}	0.3852	295.9516	1.0041	33.5128×10^{-3}
	NR	8.9769	6.07×10^{-10}	0.3965	294.0934	0.9712	32.6538×10^{-3}
	LM	8.9323	1.20×10^{-9}	0.3884	625.4218	1.0003	28.2797×10^{-3}

According to Table 2, LM method successfully extracts the characteristic parameter values of single-diode models of different material modules, which proves that algorithm proposed in this paper has wide applicability. Compared with modified Newton method and Reduced-form method, evaluation parameter value RMSEI of curve-fitting algorithm proposed in this paper is smaller, that is, the result of LM method proposed in this paper is more accurate. At the same time, RF method has two parameters to be solved, while modified Newton method has three parameters to be solved. Proposed algorithm in this paper has four parameters to be solved, while the evaluation parameters are reduced in turn and the model simulation accuracy increases gradually, which shows that excessively reducing the number of parameters will reduce the simulation accuracy of the algorithm.

This paper visualizes the curve-fitting effect of LM method on monocrystalline silicon module JKM280 (as shown in Fig. 4 and Fig. 5) and polycrystalline silicon module JKM315 (as shown in Fig. 6 and Fig. 7).

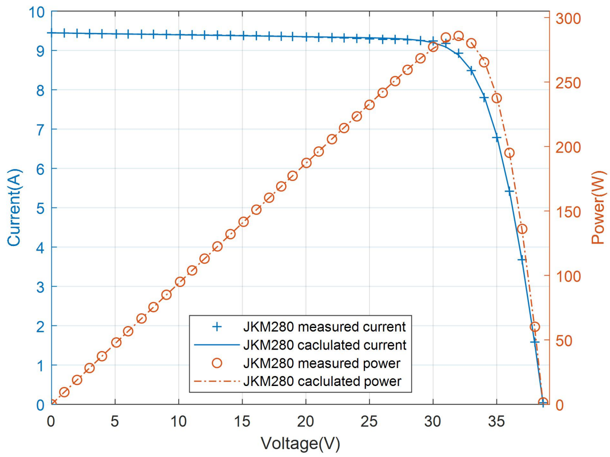

Fig. 4 Comparison between theoretical calculation and measured data of monocrystalline JKM280 PV module

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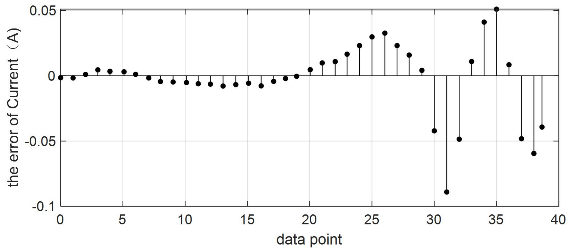

Fig. 5 Error diagram of current of monocrystalline JKM280 PV module

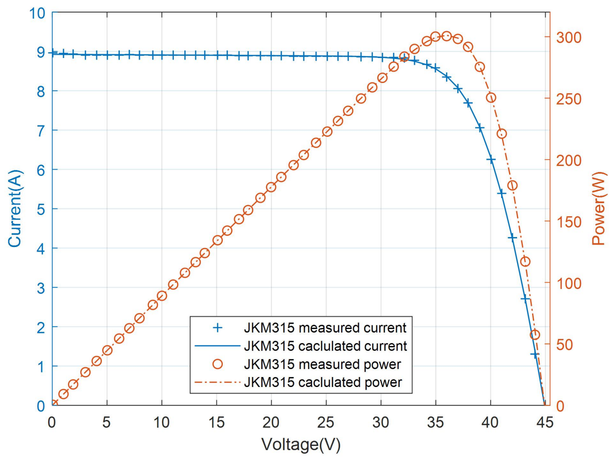

Fig. 6 Comparison between theoretical calculation and measured data of polysilicon JKM315 PV module

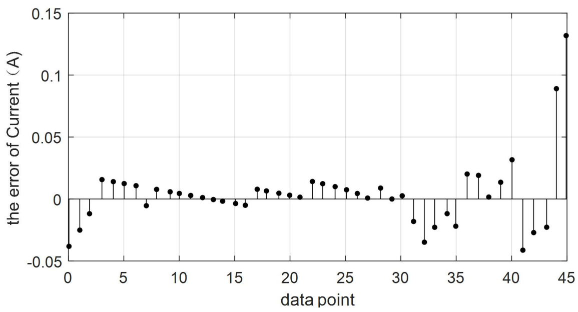

Fig. 7 The error diagram of current of polysilicon JKM315 PV module

As can be seen in Fig. 4 and Fig. 6, the theoretical output characteristic curves of current and power are consistent with the actual/measured output characteristic curves of current and power, which proves that the curve fitting algorithm in this paper can effectively converge for monocrystalline silicon and polycrystalline silicon modules, that is, the curve fitting algorithm proposed in this paper has good applicability. It can be seen from Fig. 5 and Fig.7 that the distribution of current errors of the two types of PV modules from the short-circuit point to the maximum power point, is significantly better than that from the maximum power point to the open circuit point.

In order to further analyze the current error, take the JKM315 case study data as an example, visualize the current error distribution of the two curve fitting algorithms in box diagram, as shown in Fig. 8.

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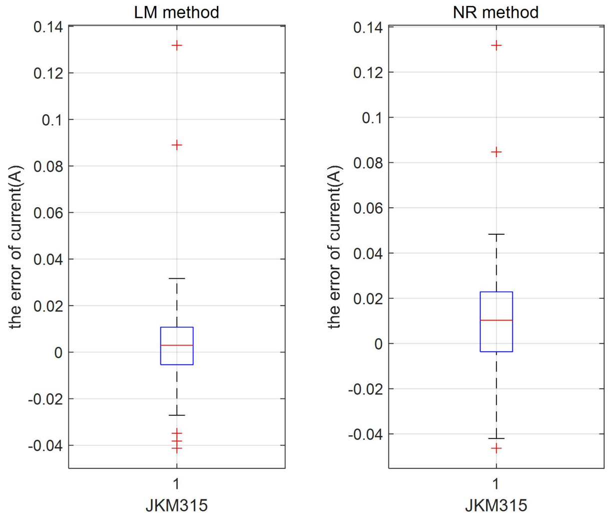

Fig. 8 Boxplot diagram of current error of JKM315 photovoltaic module

As shown in Fig.8, it can be clearly observed that the current errors of the two Curve-fitting algorithms contain abnormal points (red points). The current error of curve fitting LM method is evenly and symmetrically distributed near the zero point, whether at the edge or 1/4 line (Blue rectangle), while the current error distribution range of NR method is relatively large, and the rectangular symmetry center deviates significantly from the zero line. Therefore, it can be judged that the current error distribution of the curve fitting algorithm proposed in this paper is more applicability.

V. Conclusions

This paper propose a curve-fitting algorithm based on the LM method. In order to avoid matrix singularity and other matrix problems, open circuit point is used to express diode reverse saturation current so that the characteristic parameters of the single-diode model of PV can be successfully and accurately extracted by the algorithm. At the same time, in order to balance the accuracy and solving speed and complexity, fail to excessive reduction of the algorithm solving dimension, four suitable characteristic parameters are selected as the parameters vector to be sought. At last, results of curve-fitting algorithm of dimensionality reduction method and modified Newton method is compared with the proposed method. It has proved that LM method has high simulation accuracy and wide applicability for single-diode model of PV. With this method, single-diode model can describe the electrical characteristics of photovoltaic modules well, and provides a great referential significance for photovoltaic system designers.

Acknowledgments

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