Asymptotic Analysis of Solutions of Higher Order Differential Equations and Higher Order Difference Equations

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Abstract

The periodicity and vibration of the almost periodic solutions of difference equations and higher-order differential equations are also helpful to the establishment of today's ecological mathematical models, which have made important theoretical contributions to the analysis and discussion of the current severe environmental problems and achieved rich academic achievements. The boundedness of solutions of higher-order differential equations was first proposed in the study of biology, ecology, physiology, physics and neural networks, and it is a very important field in the study of differential equations. Difference equation is also called discrete dynamical system, and it is a powerful mathematical tool in the fields of science, technology and economy. It is one of the important means to study higher-order differential equations. Based on the relevant theoretical knowledge of difference equations and higher-order differential equations, under the premise of exponential dichotomy, using the definition and basic properties of asymptotic almost periodic sequence and the principle of contractive mapping, this paper proves the existence of asymptotic almost periodic solutions for a class of Quasilinear Difference Equations.

Keywords: *Keywords: higher order differential equation, Higher order difference equation, Asymptotic behavior of solution*

Ⅰ. Introduction

With the development of science and technology, modern machinery and information engineering are constantly updated and developed. People begin to rely more and more on various advanced and efficient mathematical processing methods [1]. Among them, the proposal and preliminary application of almost periodic function have been realized as early as last century. As we all know, many important dynamical systems are described by differential equations or difference equations [2]. Difference equation is a mathematical theory parallel to differential equation. The difference equation reflects the value and change law of discrete variables, which is to introduce discrete variables in the system or process for the goal to be solved [3-4]. Difference equations also appear in the discretization of differential equations [5]. The behavior analysis of solutions is the main research problem in the qualitative theory of differential equations and difference equations. The description of the system by ordinary differential equations is based on the basic assumption that the future of the system is only related to the current state and has nothing to do with the past [6]. In the definite solution of differential equations, including difference equations, besides the initial value problem, there is also a class of boundary value problems and eigenvalue problems which are closely related to mathematical physics problems. Up to now, this problem has made great progress in the depth and breadth of problems and research methods [7-8].

The periodicity and oscillation of almost periodic solutions of difference equations and differential equations also help to establish today's ecological mathematical model, make important theoretical contributions to the analysis and discussion of current severe environmental problems, and obtain rich academic achievements. As an important mathematical model to describe these systems, the research of differential difference equations, especially the oscillation of differential difference equations, has developed rapidly [9-10]. With the rapid development of science and technology, it not only plays a more and more important role in many cutting-edge fields, but also becomes an indispensable mathematical tool in the fields of biology, chemistry, modern physics and so on [11]. As a part of qualitative theory, the vibration theory of differential equations and difference equations has developed rapidly in recent 30 years and has become one of the hot topics in current research [12]. The boundedness of solutions of ordinary differential equations was first put forward in the study of biology, ecology, physiology, physics and neural network, which is a very important field in the study of differential equations. Difference equation is also called discrete dynamical system, and it is a powerful mathematical tool in the fields of science, technology and economy. On the one hand, it is one of the important means to study differential equations [13]. In the past decades, a large number of delay differential equation models have been proposed and used to describe the research objects in many fields such as ecology, physics, chemistry, engineering, medicine, etc. The research on the dynamic behavior of these mathematical models has important practical significance and application prospects.

Ⅱ. Asymptotic Almost Periodic Solutions of Difference Equations

A. Basic Concept

In this paper, we use ruler to represent the set of real numbers and Z to represent the set of integers. C (R) is a Banach space composed of all bounded complex valued continuous functions on the R-Scale, and X represents a Banach space in which the norms are supremum norms.

Definition (1) as a set of r ruler sets, if p is called relatively thick in r, it means that l>0 is satisfied

$$[a,a+l]\cap P\neq\Phi,\forall a\in R \quad (1)$$

Definition (2) call the function f (t) almost periodic, that is, occupy a translation set for any ε > 0, f (T)

$$T(f,\varepsilon)=\{\tau\in R:|f(t+\tau)-f(t)|<\varepsilon,\ \forall t\in R\} \quad (2)$$

Is relatively thick on R .. The whole of such a function is expressed by AP(R)

Definition (3) call sequence $x:Z\to X$ almost periodic sequence, that is, ε translation set for any given $\varepsilon>0$, X

$$T(x,\varepsilon)=\{\tau\in Z:\|x(n+\tau)-x(n)\|<\varepsilon,\ \forall n\in Z\} \quad (3)$$

Is relatively thick in Z .. The set of such sequences is represented by AP(Z, x), and τ is called ε - translation number of x.

Definition (4) call the function f ∈ C (R) asymptotically almost periodic, that is, if $f=g+\phi$, where g ∈ AP (R), $\phi\in C_0(R)$, G is the almost periodic part of F. All of such functions are represented by AAP (R)

Among

$$C_0(R)=\left\{\phi\in C(R):\lim_{|t\to\infty|}|\phi(t)|=0\right\} \quad (4)$$

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Definition (5): Call sequence $x: Z \to X$ asymptotically almost periodic sequence, which means that if $x = x_1 + x_2$, Xl ∈ AP(Z, x) and x_2 satisfy $\|x_2\| \to 0, n \to \infty$, AAP(Z, x) is used to represent the set of such sequences

In definition (6), NxN matrix A (n) = (aij (n)) NxN is called an almost periodic matrix sequence, which means that all elements aij (n) of matrix A (n) are almost periodic sequences

Definition (7): consider the function f (n, x): $Z \times \Omega \to R^N$, Ω are compact sets in RN, and k is an arbitrary compact subset in Ω. For any $\varepsilon > 0$, if there is No ∈ N, for any m ∈ Z, there is $\tau \in \{m, m+1, ..., m+N_0\}$, so that

$$\sup_{(n,\alpha)\in Z\times K} \left| F(n+\tau,\alpha) - F(n,\alpha) \le \varepsilon, \forall n \right| \in Z \quad (7)$$

Then f (n, x) is called uniformly almost periodic for $x \in \Omega$ with respect to n. Denoted as f ∈ APU (Z, Ω, RN)

Definition (8) bounded functions $F: Z \times \Omega \to R^N$ and Ω are defined as compact sets in RN. if $F = G + \varphi$, $G \in APU(Z \times \Omega, R^N)$ and $\Phi \in C_0(Z \times \Omega, R^N)$, f is called uniformly asymptotically almost periodic function. among

(8)

Consider equation

$$x(n+1) = A(n)x(n), n \in Z \quad (9)$$

$$x(n+1) = A(n)x(n) + b(n), n \in Z \quad (10)$$

A(n) is a reversible almost periodic matrix sequence, $b(n) \in AAP(Z, R^N)$

Definition (11). On Z, equation (9) has exponential dichotomy, that is, if there is a projection P and constants k > 0, α > 0, the existence of any given m, n ∈ Z

$$\begin{aligned} &\left|X(n)PX^{-1}(m)\right| \le Ke^{-\alpha(n-m)}, n \ge m \\ &\left|X(n)(I-P)X^{-1}(m)\right| \le Ke^{-\alpha(m-n)}, m \ge n \end{aligned} \quad (11)$$

Where x (n) is the basic solution matrix of equation (2-1)

B. Related Lemmas

Equation (9) has exponential diversity in z, that is, if there are projections P1, P2, P3, PiPj=0,i $\neq$ j, P1+P2+P3=I and constants K>0, α >0, then any given m, n∈Z exists.

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$\left|X(n)P_1X^{-1}(m)\right| \leq Ke^{-\alpha(n-m)}n \geq m$ (10)

$\left|X(n)P_2X^{-1}(m)\right| \leq Ke^{-\alpha(m-n)}, m \geq n$ (11)

$\left|X(n)P_3X^{-1}(m)\right| \leq \begin{cases} Ke^{-\alpha(n-m)}, 0 \leq m \leq n \\ Ke^{-\alpha(m-n)}, n \leq m \leq 0 \end{cases}$ (12)

If and only if equation (9) has exponential dichotomy on Z and P3 = 0

Makes A(n) a reversible almost periodic matrix sequence, and assumes that the homogeneous form (10) corresponding to Equation (9) has exponential diversity on Z, and for any given b(n)∈AAP(Z, RN), Equation (10) has a solution $\phi_0(n) \in AAP(Z, R^N)$. If =0, the solution is unique

$f(n) = g(n) + \phi(n)$ (13)

Where g ∈ AP (Z, RN), $\phi \in C_0(Z, R^N)$. be

(14)

Thus

$\|f\| \geq \|g\| \geq \inf_{n\in Z} |g(n)| \geq \inf_{n\in Z} |f(n)|$ (15)

Let A (n) be a reversible almost periodic matrix sequence. Assuming that the homogeneous formal equation (10) corresponding to equation (9) on Z has exponential dichotomy, then for any given asymptotically almost periodic sequence b (n), equation (10) has a unique asymptotically almost periodic solution

By definition (11) and lemma 1, it is easy to deduce that theorem 1 is obviously true.

Ⅲ. Asymptotically Almost Periodic Solutions of Differential Equations

A. Basic Concept

In recent years, due to the high application value of neutral differential equations in real life, it is becoming more and more important to study it theoretically, including the oscillation and asymptotic behavior of neutral differential equations. In the existing results, for the first order linear neutral differential equations

$\left[x(t) + px(t-\tau)\right]' + Q_1(t)x(t-\sigma_2) = 0$ (16)

There are many discussions on P ∈ (0,1)

Because it is difficult to analyze, among the existing results, there are few studies on higher-order nonlinear neutral differential equations. In this chapter, we consider a class of higher-order nonlinear neutral differential equations.

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Under a wide range of conditions, by using some important differential inequalities, we obtain sufficient conditions for oscillation of this equation. Considering Higher Order Philippine Linear Neutral Differential Equations.

$$\left(x(t)-p(t)x(t-\tau)\right)^{(n)}+f\left(t,x\left(\sigma(t)\right)\right)=0 \quad (17)$$

Where n is an odd number, and $n\geq 3,\tau>0$, P, $\sigma\in C\left(\left(t_0,\infty\right),R^+\right),\sigma(t)\leq t,\lim_{t\to\infty}\sigma(t)=\infty, f\in C\left(t_0,\infty\right)\times R,R\left.\right), f\left(t,u\right)$ On u non subtraction

In addition to the above conditions, p and f satisfy the following conditions:

Or there is t '≥ to, so that

$$\liminf_{m\to\infty}\prod_{i=1}^{m}p\left(t^{'}+i\tau\right)<\infty \quad (18)$$

Or for any t≥to, $\lim_{m\to\infty}\prod_{i=1}^{m}p\left(t+i\tau\right)=\infty$, but there is t≥to, so that

$$\sum_{m=1}^{\infty}\prod_{i=1}^{m}\frac{1}{p\left(t'+i\tau\right)}=\infty \quad (19)$$

(II)There are H > 0 and functions g (U), q (T), so that for t ≥ to, $0<|u|\leq H$, there is

$$\frac{f\left(t,u\right)}{u}\geq q(t)g(u)>0 \quad (20)$$

In which $q\in C\left(t_0,\infty\right),R^+,g\in C\left(R,R^+\right)$, and when 0 < u ≤ h, g(u) is a decreasing function

(III) $g(-u)=g(u),\lim_{u\to 0}g(u)=1$,and

$$\int_{\alpha}^{\infty}\left(1-g\left(e^{-u}\right)\right)du<\infty \quad (21)$$

(IV)For any $j\in\left\{0,2,4,\ldots,n=1\right\}$ and $d\neq 0$, the following equation holds:

$$\left|\int_{t_0}^{\infty}s^{n-j-1}d\left(s,d\left(\sigma(s)\right)^{j-1}\right)ds=\infty\right| \quad (22)$$

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For the convenience of discussion, in this chapter, we always assume that $\sigma(t)$ is strictly single increment on $\left[t_{0,}\infty\right]$

B. Relevance Lemma

Assumes that n is an odd number, y(t) is a differentiable function of degree n, y(t)>0, y(n)(t)≤0 and y(n)(t) is not always equal to zero. There is an even number k, so that

$$y^{(i)}(t)>0,\ \ i=0,1,2,...,k$$

$$(-1)^i y^{(i)}(t)>0,\ \ i=k+1,k+2,...n-1 \text{ (23)}$$

$$\lim_{t\to\infty} y^{(i)}(t)=0, i=k+1,k+2,...n-1$$

Found

In addition to satisfying the conditions in lemma 23, if y (t) is bounded, then k = 0 in lemma (23), i.e

$$(-1)^i y^{(i)}(t)>0,\ \ i=1,2,...,n-1 \text{ (24)}$$

Established. in addition

$$\lim_{t\to\infty} y^{(i)}(t)=0, i=1,2,...,n-1 \text{ (25)}$$

And for any $0<\tau(t)<t$, the following inequality

(26)

Finally established

It is proved that if not, assuming K ≥ 2, at least two adjacent terms in all derivatives of Y (T) are positive, which will lead to the contradiction between the boundedness of $\lim y(t)=\infty$ and Y (T), so k = 0

If $i\in\{1,2,...,n-1\}$ exists, $\lim_{t\to\infty} y^{(i)}(t)\neq 0$ will also lead to $\lim_{t\to\infty} y(t)=\infty$, which still contradicts the boundedness of y(t), so formula (25) holds

From the Taiylor expansion of $y'(\tau(t))$, we get

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$$y^{'}\left(\tau\left(t\right)\right)=y^{'}\left(t\right)+y^{"}\left(t\right)\left(\tau\left(t\right)-t\right)+\frac{y^{"}\left(t\right)}{2!}\left(\tau\left(t\right)-t\right)^{2}+...+\frac{\left(\tau\left(t\right)-t\right)^{n-1}}{\left(n-1\right)!}y^{(n)}\left(t\right)+R_{n}\left(t\right)$$

In which $R_n\left(t\right)$ is lagrange type remainder

Because of $y^{'}\left(t\right)<0$, $y^{"}\left(t\right)>0$, $\tau\left(t\right)<t$, the formula (3.2.3) is finally established

Suppose condition (I) holds, X (T) is the final positive solution of equation (23), let $y\left(t\right)=x\left(t\right)-p\left(t\right)x\left(t-\tau\right)$, then y (T) > 0 holds

It is proved that by equation (3.1.1) and y(n)(t)≤0, y(n)(t)≤0 and y(n)(t) is not always equal to zero. By lemma 3. 2. 1, know $y^{"}\left(t\right)>0$. Therefore, y(t) is the final number. We assert that y (t) > 0 is finally true

In fact, if not, y (T) < 0 is finally established, then there must be y '(T) < 0, otherwise $\lim_{t\to\infty} y\left(t\right)=+\infty$ is in contradiction with y (T) less than 0. Then there are $\alpha<0$ and T1 ≥ T0, so that when t ≥ TL, there is $y\left(t\right)\le\alpha<0$. be

$$x\left(t\right)\le\alpha+p\left(t\right)x\left(t-\tau\right),t\ge t_1 \quad (27)$$

For any given t*, let ti=t*+iT,i=0,1, ..., and get by induction

$$x\left(t_n\right)\le\alpha+p\left(t_n\right)x\left(t_{n-1}\right)\le\alpha\left(1+\sum_{i=k+2}^{n}\prod_{j=0}^{n-i}p\left(t_{n-j}\right)\right)+\sum_{i=k+1}^{n}p\left(t_i\right)x\left(t_k\right) \quad (28)$$

If t '≥ t0 exists, so that (3.1.2) holds, there are the following two cases, that is

(29)

$$\lim_{n\to\infty}\sum_{i=k+2}^{n}\prod_{j=0}^{n-i}p\left(t_{n-j}\right)=\infty \quad (30)$$

If $\lim_{n\to\infty}\sum_{i=k+2}^{n}\prod_{j=0}^{n-i}p\left(t_{n-j}\right)<\infty$ is established, $\lim_{n\to\infty}\prod_{j=k+1}^{n}p\left(t_i\right)=0$, thus $x\left(t_n\right)\le\frac{\alpha}{2}<0$, is finally established, which contradicts x(t)>0

If $\lim_{n\to\infty}\sum_{i=k+2}^{n}\prod_{j=0}^{n-i}p\left(t_{n-j}\right)=\infty$ is established, $\lim_{n\to\infty}x\left(t_n\right)=-\infty$ is also in contradiction with X (T) > 0

If formula (3). 1. 2) is not true, then (3). 1. 3) Establishment. Let $d_n = \prod_{i=k+1}^{n} p(t_i)$

By (3. 2. 4), get

$$x(t_n) \le \alpha \left(\sum_{i=1}^{n} \prod_{j=1}^{i} \frac{1}{p(t_j)} - \sum_{i=1}^{k+1} \prod_{j=1}^{i} \frac{1}{p(t_j)} \right) \prod_{i=1}^{n} p(t_i) + d_n x(t_k) \quad (31)$$

$$= \alpha \left[\left(\sum_{i=1}^{n} \prod_{j=1}^{i} \frac{1}{p(t_j)} - \sum_{i=1}^{k+1} \prod_{j=1}^{i} \frac{1}{p(t_j)} \right) \prod_{i=1}^{k} p(t_i) + \frac{x(t_k)}{\alpha} \right] d_n \quad (32)$$

$\lim_{n\to\infty} x(t_n) = -\infty$ is obtained from (3.1.3) $\alpha < 0$, which is still in contradiction with X (T) > 0

So there must be y (t) > 0

Ⅳ. Conclusions

As one of the extended forms of periodic functions, almost periodic functions can form Banach spaces under the supremum norm. Therefore, it has more theoretical research value than periodic function. Asymptotic periodic function is a kind of almost periodic function which is widely used up to now. It has some basic properties and a specific representation of almost periodic function. For the study of oscillation of higher order differential-difference equations, we will focus on the case of multiple delays in the future. With the increase of delays, the difficulties will increase and more mathematical tools need to be used. In my study and work, I will continue to think and further study the oscillation theory of higher-order differential difference equations. The research on the asymptotic almost periodic function is mainly to apply it to the equation, that is, to find the asymptotic almost periodic solution of the equation, and then to solve the specific practical problems by establishing a mathematical model.

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