# Design of Quad-rotor Controller Based on Fractional Order Sliding Mode Control

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## Abstract

According to the chattering problems of traditional sliding mode index exponential reaching law, this paper proposes a fractional order sliding mode index exponential reaching law control strategy, which is applied to the quad-rotor helicopter attitude control. Combined the theory of fractional order calculus and sliding mode variable structure control theory, the fractional order sliding mode controller is designed. The method of lyapunov analysis proves that this controller can make the system asymptotically stable. Simulation and experiments show that the proposed fractional order sliding mode control system not only undermines the chattering of traditional sliding mode exponential reaching law but also reduces the adjusting time and control margin of the system.

Keywords: fractional order, sliding mode control, chattering, quad-rotor.

## I. Introduction

With the continuous development of aerospace technology, quad-rotor helicopter are largely applied on military and civilian [1-3]. At present, quadrotor helicopters are being widely used in agriculture to realize accurate agricultural production. Obtaining timely, effective and accurate farmland land and environmental information is the key to accurate agricultural production. Signal acquisition of farmland information by quadrotor helicopters can obtain large-scale farms and farmland land and environmental information, which facilitates the formulation of variable operations in the field and subsequent field management arrangements.

Quad-rotor helicopter control system itself has characteristics as multi-variable, nonlinear, higher order, strong coupling and more sensitive to interference, and in the entire field of control engineering, it is a relatively complex control object. Attitude control of quad-rotor helicopter is the key in the flight controls. At present, quad-rotor helicopter flight control have a lot of control algorithms, such as classic PID algorithm, fuzzy algorithms, neural algorithm, adaptive algorithm and sliding mode etc [4-7]. Variable structure control system have steady-state invariance to the perturbations and external disturbances, but whose own chattering, and most of integer order control limited its development [8-10]. In recent years, the fractional order variable structure control is increasingly used in the theory of variable structure control system and achieved some research results [11-13]. For the chattering of traditional integer order sliding controller, a fuzzy sliding mode controller based on fractional order is introduced[14], which objects permanent magnet synchronous motor . In order to make flexible spacecraft with a fast track and robustness, a fractional order sliding mode controller is proposed [15]. To the attitude control of flying wing UAV on interference, sliding surface is designed [16], reducing the overshoot ordinary integral sliding surface.

The paper proposes a fractional order sliding mode reaching law based on the traditional sliding mode reaching law, which combines fractional calculus and variable structure control, uses lyapunov analysis, proves the convergence of the fractional sliding mode reaching law. To the quad-rotor helicopter attitude control models, a fractional sliding mode controller is designed. The simulation demonstrate the effectiveness of the control strategy based on the object of quad-rotor helicopter attitude control model.

## II. Three degrees freedom of quad-rotor helicopter system modeling

This paper objects three degrees freedom of quad-rotor helicopter on independent research in Zhou Kou Normal College, study quad-rotor helicopter attitude control that consists of four rotor helicopter body and control system. Among the system, four propellers were cross structure, divided into two groups between left or right and front or back.

#### 2.1 Dynamic Model

Ignoring the impact on the atmosphere propeller, the gyroscopic effect and the friction between the bearings, the force of the system is shown in Fig 1,  $V_f$ ,  $V_b$ ,  $V_l$ ,  $V_r$  acting on the front, rear, left, and right propellers voltage to the motor respectively, defining the Y axis as the pitch axis of the body, the X axis as the roll axis, the Z axis as the yaw axis. Front and rear propeller control the pitch angle, left and right propeller control the roll angle, four propellers control the yaw angle.



Fig 1: Dynamics of 3 DOF Hovering System

Suppose the roll angle as zero, the lift front and rear propeller pitch angle generated control pitch angle torque, equations of motion for the pitch angle can be obtained.

$$J_{p} \not = (F_{f} - F_{b})l \tag{1}$$

$$F_f - F_b = (V_f - V_b)K_f \tag{2}$$

By equation (1), (2) to the availability of equation (3):

$$\mathbf{p} = (V_f - V_b) l \frac{K_f}{J_p}$$
(3)

Where  $F_f$ ,  $F_b$  as the lift generated by the front and rear propeller,  $V_f$ ,  $V_b$  as the motor voltage of the front and rear propeller,  $J_p$  as the moment inertia of pitch axis, l as the distance from the center between the pitch axis and the propeller shaft, p as the pitch axis,  $K_f$  as the lift coefficient.

Similar to the pitch angle, the equation of motion for the roll angle:

$$\mathbf{K} = (V_r - V_l) l \frac{K_f}{J_r} \tag{4}$$

Where  $V_r$ ,  $V_l$  as the motor voltage of the left and right propeller, r as the roll angle,  $J_r$  as the moment inertia of the roll axis.

When front, rear, left, right four propellers rotate, the torque will be generated respectively as  $T_f$ ,  $T_b$ ,  $T_l$ ,  $T_r$ , the equation of motion for the yaw angle:

$$J_{y} \mathcal{B} = \sum T = T_{f} + T_{b} + T_{l} + T_{r}$$

$$\tag{5}$$

$$J_{y} \mathscr{B} = \sum T = K_{t,c} (V_{f} + V_{b}) + K_{t,n} (V_{r} + V_{l})$$
(6)

$$K_{t,c} = \frac{K_{t,c}}{J_{y}} (V_{f} + V_{b}) + \frac{K_{t,n}}{J_{y}} (V_{r} + V_{l})$$
(7)

Where  $J_y$  as the moment inertia of the yaw axis, y as the yaw angle,  $K_{t,c}$  as the counterclockwise torque,  $K_{t,n}$  as the clockwise torque, and  $K_{t,c} = -K_{t,n}$ .

#### 2.2 State space equation

By equation (2), (3),(6) to the availability of equation (8),the state space equation of the three freedom degrees to four-rotor hover system:

$$\begin{cases}
\mathbf{x} = Ax + Bu \\
y = Cx + Du
\end{cases}$$
(8)

Where  $x^T = [y, p, r, \mathcal{K}, \mathcal{K}]$  as the state vector,  $u^T = [V_f, V_b, V_r, V_l]$  as the control input,  $y^T = [y, p, r]$  as the measuring input, A, B, C, D as the coefficient matrix. In which

The state space model shows that the system is a linear system of four input- three output.

#### III. Fractional Sliding Mode Controller

#### **3.1 Fractional Calculus**

Fractional calculus mainly study on the differential any order and integral operators characteristics and application. compared with the integer order calculus, fractional differential process and the integration process have more flexibility. With the development of computer technology, fractional calculus have been widely used in industrial processes, becoming a branch of the field of control engineering.

Usually, fractional calculus expressed as the following: 
$$_{\alpha}D_{t}^{a}f(t) = \begin{cases} \frac{d^{\alpha}}{dt^{\alpha}}f(t), \operatorname{Re}(\alpha) > 0\\ f(t), \operatorname{Re}(\alpha) = 0\\ \int_{\alpha}^{t}f(\tau)(d\tau)^{-\alpha}, \operatorname{Re}(\alpha) < 0 \end{cases}$$

Where  $_{\alpha}D_{t}^{a}$  as calculus operators,  $\alpha$  as calculus higher limit, *t* as calculus lower limit, when  $\alpha > 0$ ,  $_{\alpha}D_{t}^{a}$  as fractional differential, when  $\alpha < 0$ , as fractional integral.

This paper applies the Caputo fractional calculus which is applied commonly in the control field[17-18]:

$${}_{\alpha}D_{t}^{a}f(t) = \frac{1}{\Gamma(m-\alpha)}\int_{\alpha}^{t} \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}}d\tau$$

Where  $\Gamma(\bullet)$  as gamma function, and  $m-1 < \alpha \le m$ .

#### 3.2 Controller Design

Structuring fractional exponential reaching law as follows:

$$s^{(\alpha)} = -\varepsilon sign(s) - ks \tag{9}$$

As it is can be seen from (9), fractional sliding exponential reaching law has one more adjustable parameter than the integer order sliding mode reaching law, the parameter attributes of the system can be continuously changed. Where *s* as sliding surface,  $0 < \alpha \le 1, \varepsilon > 0, k > 0$ 

The following equation can be obtained from the equation (9):

$$\mathcal{S} = -D^{1-\alpha}(\operatorname{ssign}(s) + ks) \tag{10}$$

Select sliding surface *s* as:

$$s = Cx \tag{11}$$

Where  $C \in \mathbb{R}^{m \times n}$ , *C* is determined by using the pole assignment method, to be of the form:

Consider it in the system:

$$\mathcal{R} = Ax + Bu \tag{13}$$

where  $A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}$ 

The following equation can be obtained from the equation (12) and (13):

$$\mathcal{E} = C(Ax + Bu) \tag{14}$$

Then, the sliding mode variable structure control law is:

$$-D^{1-\alpha}(\varepsilon sign(s) + ks) = C(Ax + Bu)$$
  

$$\Rightarrow u = -(CB)^{-1}(CAx + \varepsilon D^{1-\alpha}sign(s) + kD^{1-\alpha}s)$$
(15)

Where  $k, \varepsilon, \alpha$  is three adjustable parameters. One more adjustable parameter is appeared by using traditional exponential reaching law designed controller, so the control object can be more flexible controlled by the controller.

## **3.3 Stability Analysis**

Select Lyapunov function as follows:

$$v = \frac{1}{2}s^{\mathrm{T}}s \tag{16}$$

The equation (16) is transformed as follows:

$$\boldsymbol{k} = \boldsymbol{s}^{\mathrm{T}} \boldsymbol{k} = \boldsymbol{s}^{\mathrm{T}} [-\boldsymbol{D}^{1-\alpha} (\varepsilon sign(s) + ks)]$$
<sup>(17)</sup>

In document [15], there is

$$sign(D^{1-\alpha}(-\varepsilon sign(s))) = -\varepsilon sign(s)$$
<sup>(18)</sup>

so the following equation can be obtained from the equation (17) and (18):

$$sign(\mathbf{A} = sign(s^{T})sign[-D^{1-\alpha}(\varepsilon sign(s) + ks)]$$
  
= sign(s^{T})(-\varepsilon sign(s)) + sign(s^{T})(sign(-ks)) (19)  
= -\varepsilon - k

As we can see from the equation (19):

When  $\varepsilon > 0$ , k > 0, then  $sign(\Re < 0$ , the result  $\Re < 0$  is inferred, it means that the system is asymptotic stability.

## **IV. Simulation Results**

Against the quad-rotor helicopter control object, it is expected the settling time of sliding mode structure control from the initial state  $x_0 = \{10, 4, -4, 2, 2, 1\}$  to origin less than 2 seconds.

System parameters for the quad-rotor helicopter:  $K_{t,n} = 0.0036N \cdot m/V$ ,  $K_{t,c} = -0.0036N \cdot m/V$ ,  $K_f = 0.1188N/V$ , l = 0.197m,  $J_y = 0.110kg \cdot m^2$ ,  $J_p = J_r = 0.0552kg \cdot m^2$ .

The three degrees of freedom for the quad-rotor helicopter model:

$$\mathbf{x} = \begin{bmatrix} 0 & I_{3\times3} \\ 0 & 0 \end{bmatrix}_{6\times6} x + \begin{bmatrix} 0 & 0_{3\times3} & 0_{3\times3} \\ 0.0327 & 0 & 0 \\ 0 & 0.4260 & 0 \\ 0 & 0 & 0.4260 \end{bmatrix} u$$
$$y = \begin{bmatrix} I_{3\times3} & 0 \end{bmatrix}_{3\times6} x$$

According to performance requirements, select the set of poles as  $\{-5, -5, -5\}$ .

The nominal equations of the system as follows:  $\mathbf{x} = (A_{11} - A_{12}K)x_1$ 

In which  $A_{11} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ ,  $A_{11} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  By using the pole assignment method, choose K to

make  $A_{11} - A_{12}K$  characteristic root as desirable characteristic root.

Then parameter matrix of the sliding variable structure:  $C = \begin{bmatrix} K & I_m \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 & 1 & 0 & 0 \\ 0 & 5 & 0 & 0 & 1 & 0 \\ 0 & 0 & 5 & 0 & 0 & 1 \end{bmatrix}$ 

In which  $\varepsilon = 0.01$ ,  $\alpha = 0.9$ , k = 1.

Compared the fractional sliding exponential reaching law with the traditional sliding exponential reaching law about the mode control methods, the simulation results are shown in Figs 2, 3, 4, and 5, Fig 2 is the states responses for the traditional exponential reaching law, Fig 3 is the states responses for the fractional order sliding mode index exponential reaching law, Fig 4 is the responses of the controller for the traditional exponential reaching law, Fig 5 is the responses of the controller for the fractional order sliding mode index exponential reaching law, from Figs 2, 3, 4, and 5 it can be seen, the states of the systems converge to a small region of the origin, and the chattering is reduced, and amplitude of control law is decreased effectively by using the fractional order sliding mode index exponential reaching law.



Fig 2: The states responses for the traditional exponential reaching law



Fig 3: The states responses for the fractional order sliding mode index exponential reaching law



Fig 4: The responses of the controller for the traditional exponential reaching law



Fig 5: The responses of the controller for the fractional order sliding mode index exponential reaching law

# V. Conclusion

This paper objects three degrees freedom of quad-rotor helicopter on independent research in Zhou Kou Normal College, design the controller based on fractional order sliding mode exponential reaching law against the attitude control on the subject. Appling this controller, the chattering is reduced by traditional sliding exponential reaching law control method, the adjusting time and amplitude control of the system are also reduced, improving dynamic performance of the quad-motor helicopter. The simulation results show the control strategy is effective.

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