# Effective Tracking Control of Desired Trajectory of a Manipulator Based on Impedance Control and Fuzzy Adaptive Technology

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***Abstract***

*A finite-time output constrained impedance control method for mechanical arm based on instruction filtering was proposed. The interaction between the mechanical arm and the environment was solved by impedance control technology so that the mechanical arm could track the desired trajectory. The response speed of the mechanical arm was improved by finite-time control, the tracking error was reduced, and the obstacle Lyapunov function was introduced to constrain the output state of the mechanical arm.The fuzzy adaptive technology was used to deal with unknown friction and external disturbance in the mechanical arm system. The simulation results showed that the proposed method could effectively track the desired trajectory, and the output states of the mechanical were restricted in the predefined constraint space, so it had a faster response speed and better tracking effect.*

***Keywords**: Mechanical arm, Impedance control, Fuzzy adaptive technology, Track Tracking*

## I. Introduction

In recent years, the application of mechanical arm technology has become more and more popular, and its structure has become more complicated and precise. Therefore, in order to ensure its safety and compliance, force/position control methods with higher precision are urgently needed. Mechanical arm force/position control has become an important research direction nowadays. To realize the fast tracking control of the mechanical arm, XUE C Q et al added the finite-time control technology to the fuzzy adaptive impedance control, so that the output signal of the mechanical arm system in the finite time tended to the desired signal, which greatly improved the dynamic performance of the system while speeding up the response speed.The finite-time control method has a lot of advantages, such as short setting time and strong anti-jamming ability, so it has been widely used in practical applications. Compared with the non-finite time controller, the controller using finite-time control technology can achieve better robustness and anti-interference. However, when the mechanical arm runs in an unknown or complex environment, it is necessary to ensure that the output state of the mechanical arm is limited within a given bounded interval; otherwise, the mechanical arm may be damaged. The mechanical arm control methods proposed in the literature cannot guarantee that the output constraints of the mechanical arm during its operation are within a finiteinterval.

All the above methods adopt backstepping method to design the controller, but in the process of design the controller with the backstepping method, the repeated derivation of the virtual control law increases the computational complexity. To solve this problem, in this paper, the instruction filtering error compensation technology was introduced, which solves the filtering error problem by compensating signals and reduces the complexity of the controller design.Based on the above problems, in this paper, the obstacle Lyapunov function and the finite-time control method were combined with the impedance control method to propose the finite- time output constraint fuzzy adaptive impedance control applied to the mechanical arm. Finally, the simulation results proved the feasibility and effectiveness of the controller of the mechanical arm system designed in this paper.

**II. Mathematical Model and Preliminary Transformation of Mechanical Arm**

The dynamics equation of the mechanical arm is as follows:

$$D(q)\ \ddot{q}+C(q,\dot{q})\dot{q}+G(q)=\tau-\tau_f-J^T\tau_d-J^TF_e \tag{1}$$

Where $D(q)\in R^{n\times n}$ is the inertia matrix of the mechanical arm; $C(q,\dot{q})\in R^{n\times n}$ is the centrifugal force and Coriolis force matrix of the mechanical arm; $G(q)\in R^{n\times 1}$ is the gravity term vector of the mechanical arm; $\tau\in R^{n\times 1}$ is the torque vector of each joint of the mechanical arm; $\tau_f\in R^{n\times 1}$ is the friction vector subjected to the mechanical arm; $\tau_d\in R^{m\times 1}$ is the external interference vector of the mechanical arm; $J\in R^{m\times n}$ is the Jacobian matrix of the mechanical arm; $F_e\in R^{m\times 1}$ is the contact force exerted by the environment on the end of the mechanical arm; n is the degree of freedom of the mechanical arm.

The kinematic equation of the mechanical armis as follows:

$$x=\varphi(q),\dot{x}=J(q)\dot{q},\ddot{x}=J(q)\ \ddot{q}+\dot{J}(q)\dot{q}$$

By converting the above relation, the following equation can be obtained:

$$\dot{q}=J^{-1}(q)\dot{x},\ddot{q}=J^{-1}(q)\ \ddot{x}-J^{-1}(q)\ \dot{J}(q)J^{-1}(q)\dot{x} \tag{2}$$

Where $x\in R^{m\times 1}$ is the coordinate of the end of the mechanical arm in the Cartesian coordinate system,m is the dimension of the motion space of the mechanical arm; $q\in R^{n\times 1}$ is the angle of the joints of the mechanical arm
The impedance control relation between the mechanical arm end position and end force [17]

$$F_e=M_d\ddot{E}\ +B_d\dot{E}+K_dE \tag{3}$$

Where $E=x_d-x$ , $x_d$ is the command track; $M_d$is the expected inertia matrix of the mechanical arm; $B_d$is the expected damping matrix of the mechanical arm; $K_d$is the expected rigidity matrix of the mechanical arm. When the end of the mechanical arm moves in free space,$F_e$=0.

If x precisely follows the expected trajectory $x_r\in R^{m\times 1}$, then Equation (3) can be written as follows

$$F_e=M_d\left(\ddot{x}_d-\ddot{x}_r\right)\ +B_d\left(\dot{x}_d-\dot{x}_r\right)+K_d\left(x_d-x_r\right) \tag{4}$$

According to Equation (4), if $x_d$, $M_d$, $B_d$, *and* $K_d$ are known, the expected trajectory x can be obtained through Equation (4).

SubstitutingEquation (2) into Equation (1) , the following equation can be obtained

$$D_x\ddot{x}\ +C_x\dot{x}+G_x=\tau_x-\tau_{fx}-\tau_d-F_e \tag{5}$$

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Where

$$\begin{cases} D_x = J^{-T} D(q) J^{-1}, G_x = J^{-1} G(q), \\ C_x = J^{-T} \left( C(q, \dot{q}) - D(q) J^{-1} \dot{J} \right) J^{-1}, \\ \tau_{fx} = J^{-T} \tau_f, \tau_x = J^{-T} \tau \end{cases}$$

To facilitate representation, the new variable is redefined as follows:

$$\begin{cases} y = x_1 = x \\ \dot{x}_1 = x_2 \\ \dot{x}_{,} = D_x^{-1} \left[ \tau_x - \tau_{fx} - \tau_d - F_e - C_x x_2 - G_x \right] \end{cases} \tag{6}$$

When the mechanical arm works in an unknown restricted space, its output statesshould be limited within the compact set $\Omega_x$, where $\Omega_x = \{|x_{1i}| \le k_{ci}, i =, \cdots, m\}$ and $k_{ci}$is a normal number. The control objective is to design the control rate $\tau_x$to make $x_1$track the desired trajectory $x_r$, and at the same time, the output state of the mechanical arm's control system be always in the given compact set $\Omega_x$.

Lemma 1 For any real number $\lambda_1>0, \lambda_2>0$ and $0<\gamma<1$,

Then the extended Lyapunov condition of finite-time stability is

$$\dot{V}(x) < -\lambda_1 V(x) - \lambda_2 V^r(x)$$

The semi-global finite time state of the system is stable when satisfying the above equation. The convergence time of the system T can be estimated as

$$T_s = t_0 + \left[ \frac{1}{\lambda_1 (1-\gamma)} \right] \log \left[ \frac{1}{\lambda_2} (\lambda_1 V^{1-\gamma}(t_0) + \lambda_2) \right]$$

Lemma 2For $x_i \in R, i = 1, 2, \cdots, n, 0 < p \le 1$ , the following inequalities are true

$$(\sum_{i=1}^{n} |x_i|)^p \le \sum_{i=1}^{n} |x_i|^p \le n^{1-p} (\sum_{i=1}^{n} |x_i|)^p$$

Lemma 3 The form of finite-time instruction filtering is:

$$\begin{cases} \dot{\eta}_1 = \varpi_1 \\ \varpi_1 = -R_1 IIsig(\eta_1 - \alpha_r) + \eta_2 \\ \dot{\eta}_2 = -R_2 sig(\eta_2 - \varpi_1) \end{cases}$$

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Where

$$II = diag(\left|\eta_{11} - \alpha_{r1}\right|^{1/2}, \cdots, \left|\eta_{1n} - \alpha_{rn}\right|^{1/2}; \eta_1 = x_{1,c};$$

$\eta_2 = \dot{x}_{1,c}; \alpha_r$ is the input signal of the filter: $\eta_1$ and $\eta_2$ are the output signal of the filter; $R_1$ and $R_2$ are filter parameters. If the input signal$\alpha_r$ satisfies $\left|\alpha_r - \alpha_{r0}\right| \le \beta$, then the following inequality is true.

$$\begin{cases} \left|\eta_1 - \alpha_{r0}\right| \le \omega_1 \beta = \Lambda_1 \\ \left|\varpi_1 - \alpha_{r0}\right| \le \psi_1 \beta^{1/2} = \Lambda_2 \end{cases}$$

Where $\omega_1$、$\psi_1$、$\Lambda_1$和$\Lambda_2$ are normal numbers. If the input signal is not affected by noise, $\eta_1 = \alpha_{r0}$ and $\varpi_1 = \dot{\alpha}_0$ can be obtained.

Hypothesis 1: The expected trajectory is $x_d(t)$ and its first derivative is $\dot{x}_d(t)$ and second derivative is $\ddot{x}_d(t)$.

xg(t)is smooth, bounded, and known. $\left\|x_d\right\| \le Y_0$, $\left\|\dot{x}_d\right\| \le Y_1$ and $\left\|\ddot{x}_d\right\| \le Y_2$, where $Y_0$, $Y_1$ and $Y_2$ are normal numbers.

Hypothesis 2: The unknown disturbance is $\tau_d$ and its first derivative $\dot{\tau}_d$ is smooth and bounded.

**III. Design of Fuzzy-time Adaptive Output Constraint Backstepping Controller**

According to the backstepping principle, the error variables can be defined as follows:

$$z_1 = x_1 - x_r, z_2 = x_2 - x_{1,c} 。$$

Where $x_r$ is the expected trajectory; $x_{1,c}$ is the output signal of the instruction filter. The filtering error compensation signal is defined as $\xi_i = z_i - v_i, i = 1, 2$.

Step 1: The obstacle Lyapunov function of the first subsystem is selected as follows:

$$V_1 = \frac{1}{2}\sum_{i=1}^{m} log \frac{k_{ai}^2}{k_{ai}^2 - v_{1i}^2} \tag{7}$$

$k_a = \left[k_{a1}, \cdots, k_{am}\right]^T$ ,the derivative of $V_1$ can be obtained as follows:

$$\dot{V}_1 = \sum_{i=1}^{m} \frac{v_{1i}}{k_{ai}^2 - v_{1i}^2} \dot{v}_{1i} = \sum_{i=1}^{m} \frac{v_{1i}}{k_{ai}^2 - v_{1i}^2} \left(z_{2i} + x_{1,ci} - \dot{x}_{ri} - \dot{\xi}_{1i}\right) \tag{8}$$

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The virtual control law $\alpha_1$ andcompensation signal $\xi_1$ are selected

$$\alpha_1 = -K_1 z_1 + \dot{x}_d - H_1 K_{v1} \tag{9}$$

$$\dot{\xi}_1 = -K_1\xi_1 + \xi_2 + \left(x_{1,c} - \alpha_1\right) - L_1 sign(\xi_1) \tag{10}$$

Where $K_{v1} = \left[\frac{z_{11}^{2\gamma-1}}{\left(k_{a1}^2 - z_{11}^2\right)^{\gamma-1}}, \cdots, \frac{z_{1m}^{2\gamma-1}}{\left(k_{am}^2 - z_{1m}^2\right)^{\gamma-1}}\right]$ ; the virtual control law $\alpha_1$ is the input of the filter;

$$K_1 = \mathrm{diag}\left[k_{11}, \cdots, k_{1m}\right] \text{ ; } H_1 = \mathrm{diag}\left[h_{11}, \cdots, h_{1m}\right];$$

$L_1 = \mathrm{diag}\left[l_{11}, \cdots, l_{1m}\right]$ are all positive definite matrices.

Substituting Equations (9) and (10) into Equation (8), the following equation can be obtained:

$$\dot{V}_1 = -\sum_{i=1}^{m} k_{1i}\left(\frac{v_{1i}^2}{k_{ai}^2 - v_{1i}^2}\right) - \dot{v}_{1i} = \sum_{i=1}^{m} h_{1i}\left(\frac{v_{1i}^2}{k_{ai}^2 - v_{1i}^2}\right)^{\gamma} + \sum_{i=1}^{m}\frac{v_{1i}^2}{k_{ai}^2 - v_{1i}^2} + \sum_{i=1}^{m}\frac{v_{1i} l_{1i}}{k_{ai}^2 - v_{1i}^2} sign(\xi_1) \tag{11}$$

Step 2: Select the Lyapunov function of the second subsystem as follows:

$$V_2 = V_1 + \frac{1}{2} v_2^T v_2 + \frac{1}{2r}\tilde{\theta}^2 \tag{12}$$

Taking the derivative of $V_2$

$$\begin{aligned}\dot{V}_2 = \dot{V}_1 + v_2^T \dot{v}_2 + \frac{1}{r}\tilde{\theta}\dot{\tilde{\theta}} = -\sum_{i=1}^{m} k_{1i}\left(\frac{v_{1i}^2}{k_{ai}^2 - v_{1i}^2}\right) - \sum_{i=1}^{m} h_{1i}\left(\frac{v_{1i}^2}{k_{ai}^2 - v_{1i}^2}\right)^{\gamma} + \sum_{i=1}^{m}\frac{v_{1i}^2}{k_{ai}^2 - v_{1i}^2} + \\ v_2^T\left[D_x^{-1}\left(\tau_x - \tau_{fx} - \tau_d - F_e - C_x x_2 - G_x\right) - \dot{x}_{1,c} - \dot{\xi}_2\right] + \frac{1}{r}\tilde{\theta}\dot{\tilde{\theta}} + \sum_{i=1}^{m}\frac{v_{1i} l_{1i}}{k_{ai}^2 - v_{1i}^2} sign(\xi_1)\end{aligned} \tag{13}$$

Define the nonlinear function $f(Z) = -D_x^{-1}\left(\tau_{fx} + \tau_d\right) = \left[f_1(Z), f_2(Z), \cdots, f_n(Z)\right]^T$, where $Z = \left[x_1^T, x_2^T, x_{1,c}^T, \dot{v}_2^T\right]^T$, according to the universal approximation theorem[21], there is always a fuzzy logic function $W_i^T S(Z)$ for any small constant $\varepsilon_i > 0$, so that the formula $f_i(Z) = W_i^T S(Z) + \delta_i(Z)$ is true, where $\delta_i(Z)$ represents the approximation error, which satisfies $|\delta_i| \le \varepsilon_i$. And because $v_2 = [v_{21}, \cdots, v_{2m}]^T$, then according to Young's inequality, the following equation can be obtained:

$$v_2^T f(Z)=v_{21}f_1(Z)+\cdots+v_{2m}f_m(Z)\le\sum_{i=1}^{m}\left[\frac{1}{2l^2}v_{2i}^2\|W_i\|^2 S^T(Z)S(Z)+\frac{1}{2}\varepsilon_i^2\right]+\frac{m}{2}l^2+\frac{1}{2}v_2^T v_2 \tag{14}$$

Where $l$ is a constant greater than 0.

Define $\theta = max\left\{\|\omega_1\|^2,\|\omega_2\|^2,\cdots,\|\omega_{\mathrm{m}}\|^2\right\}$, estimated error $\tilde{\theta}=\theta-\hat{\theta}$, Equation (14) can be written as:

$$v_2^T f(Z)\le\frac{1}{2l^2}v_2^T v_2\theta S^T(Z)S(Z)+\frac{1}{2}\sum_{i=1}^{n}\varepsilon_{\mathrm{i}}+\frac{m}{2}l^2+\frac{1}{2}v_2^T v_2 \tag{15}$$

Select real controller $\tau_x$, compensating signal $\xi_2$ and adaptive law $\dot{\hat{\theta}}$, i.e

$$\dot{\xi}_2=-K_2\xi_2-L_2 sign(\xi) \tag{16}$$

$$\dot{\hat{\theta}}=-\frac{r}{2l^2}v_2^T v_2 S^T(Z)S(Z)-\sigma\hat{\theta} \tag{17}$$

$$\tau_x=D_x\left(-K_2v_2-H_2v_k+\dot{x}_{1,c}-A-\frac{1}{2l^2}z_2\hat{\theta}S^T(Z)S(Z)-\frac{1}{2}v_2\right)+F_e+G_x+C_x x_2$$

$$v_2^T\left[D_x^{-1}\left(\tau_x-\tau_{fx}-\tau_d-F_e-C_x x_2-G_x\right)-\dot{x}_{1,c}-\dot{\xi}_2\right]+\frac{1}{r}\tilde{\theta}\dot{\hat{\theta}}+\sum_{i=1}^{m}\frac{v_{1i}l_{1i}}{k_{ai}^2-v_{1i}^2}sign(\xi_1) \tag{18}$$

Where K2, H2, L2, r, σare positive constants; $v_K=\left[v_{21}^{2\gamma-1},\cdots,v_{2m}^{2\gamma-1}\right]^T$

$$A=\left[\frac{v_{11}}{k_{a1}^2-v_{11}^2},\cdots,\frac{v_{1m}}{k_{am}^2-v_{1m}^2}\right]^T。$$

According to Lemma 2, the following equation can be obtained:

$$v_2^T v_k=[v_{21},\cdots,v_{2m}]\left[v_{21}^{2\gamma-1},\cdots,v_{2m}^{2\gamma-1}\right]^T=v_{21}^{2\gamma}+v_{22}^{2\gamma}+\cdots+v_{2m}^{2\gamma}\ge\left(\sum_{i=1}^{m}v_{2i}^2\right)^{\gamma}=\left(v_2^T v_2\right)\gamma \tag{19}$$

By substituting Equations (15) - (19) intoEquation (13), the following equation can be obtained

$$\dot{V}_2\le-\sum_{i=1}^{m}k_{1i}\left(\frac{v_{1i}^2}{k_{ai}^2-v_{1i}^2}\right)-\sum_{i=1}^{m}h_{1i}\left(\frac{v_{1i}^2}{k_{ai}^2-v_{1i}^2}\right)^{\gamma}-K_2v_2^Tv_2-H_1\left(v_2^Tv_2\right)^{\gamma}-\frac{\sigma\tilde{\theta}\hat{\theta}}{r}+\frac{1}{2}\sum_{i=1}^{m}\varepsilon_i^2+$$
$$\sum_{i=1}^{m}\frac{v_{1i}l_{1i}}{k_{ai}^2-v_{1i}^2}sign(\xi_1)+v_2^T L_2 sign(\xi_2)+\frac{m}{2}l^2 \tag{20}$$

**IV. Stability Proof**

According to Young's inequality, the following equations can be obtained

$$\tilde{\theta}\hat{\theta}=\tilde{\theta}\left(\theta-\tilde{\theta}\right)\leq-\frac{3}{4}\tilde{\theta}^{2}+\theta^{2} \tag{21}$$

$$\frac{l_{1i}v_{1i}}{k_{ai}^{2}-v_{1i}^{2}}sign(\xi_{1})\leq\frac{l_{1i}v_{1i}^{2}}{2\left(k_{ai}^{2}-v_{1i}^{2}\right)}+\frac{ml_{1i}}{2\left(k_{ai}^{2}-v_{1i}^{2}\right)} \tag{22}$$

$$L_{2}v_{2}^{T}sign(\xi_{2})\leq\frac{L_{2}}{2}v_{2}^{T}v_{2}+\frac{mL_{2}}{2} \tag{23}$$

Substituting Equations (21) - (23) into Equation (20), the following equation can be obtained

$$\dot{V}_{2}\leq-\sum_{i=1}^{m}\left(k_{1i}-\frac{l_{1i}}{2}\right)\left(\frac{v_{1i}^{2}}{k_{ai}^{2}-v_{1i}^{2}}\right)-\sum_{i=1}^{m}h_{1i}\left(\frac{v_{1i}^{2}}{k_{ai}^{2}-v_{1i}^{2}}\right)^{\gamma}-\left(K_{2}-\frac{L_{2}}{2}\right)v_{2}^{T}v_{2}-H_{1}\left(v_{2}^{T}v_{2}\right)^{\gamma}+\frac{1}{2}\sum_{i=1}^{m}\varepsilon_{i}^{2}+$$
$$\frac{m}{2}\left(l^{2}+L_{2}\right)+\sum_{i=1}^{m}\frac{ml_{1i}}{k_{ai}^{2}-v_{1i}^{2}}-\frac{3\sigma\tilde{\theta}^{2}}{4r}+\frac{\sigma\theta^{2}}{r}+\left(\frac{\sigma\tilde{\theta}^{2}}{2r}\right)^{\gamma}-\left(\frac{\sigma\tilde{\theta}^{2}}{2r}\right)^{\gamma} \tag{24}$$

If $\frac{\tilde{\sigma}\theta^{2}}{2r}\geq1$, then $\left(\frac{\tilde{\sigma}\theta^{2}}{2r}\right)^{\gamma}-\frac{\tilde{\sigma}\theta^{2}}{2r}+\frac{\sigma\theta^{2}}{2r}\leq\frac{\sigma\theta^{2}}{2r}$;

If $0<\frac{\tilde{\sigma}\theta^{2}}{2r}<1$, then $\left(\frac{\tilde{\sigma}\theta^{2}}{2r}\right)^{\gamma}-\frac{\tilde{\sigma}\theta^{2}}{2r}+\frac{\sigma\theta^{2}}{2r}\leq1+\frac{\sigma\theta^{2}}{2r}$;

By combining the above two equations, the following equation can be obtained

$$\left(\frac{\tilde{\sigma}\theta^{2}}{2r}\right)^{\gamma}-\frac{\tilde{\sigma}\theta^{2}}{2r}+\frac{\sigma\theta^{2}}{2r}\leq1+\frac{\sigma\theta^{2}}{2r} \tag{25}$$

The following conclusions have been proved.

When meeting the conditions $\left|v_{1i}\right|\leq k_{ai};,0<\gamma\leq1$, the following inequality is true:

$$\left(\log\frac{k_{1i}^{2}}{k_{ai}^{2}-v_{1i}^{2}}\right)^{\gamma}\leq\left(\frac{v_{1i}^{2}}{k_{ai}^{2}-v_{1i}^{2}}\right)^{\gamma} \tag{26}$$

Substituting Equations (25) and (26) into Equation (24), the following equation can be obtained.

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$$\dot{V}_2 \le -\sum_{i=1}^{m}\left(k_{1i}-\frac{l_{1i}}{2}\right)\left(\log\frac{k_{1i}^2}{k_{ai}^2-v_{1i}^2}\right)-\left(K_2-\frac{L_2}{2}\right)v_2^T v_2-\sum_{i=1}^{m}h_{1i}\left(\log\frac{k_{1i}^2}{k_{ai}^2-v_{1i}^2}\right)^{\gamma}-H_1\left(v_2^T v_2\right)^{\gamma}-$$
$$\frac{\sigma\tilde{\theta}^2}{4r}-\left(\frac{\sigma\tilde{\theta}^2}{2r}\right)^{\gamma}+\frac{m}{2}\left(l^2+L_2\right)-\frac{\sigma\theta^2}{r}+\frac{1}{2}\sum_{i=1}^{m}\varepsilon_i^2+\sum_{i=1}^{m}\frac{ml_{1i}}{k_{ai}^2-v_{1i}^2}+1\le -aV_2-bV_2^{\gamma}+c \qquad (27)$$

Where,

$$a=\min\left\{\min_{m=1,\cdots,m}\left(2k_{1i}-l_{1i}\right),2K_2-L_2,\frac{\sigma}{2}\right\};$$

$$b=\min\left\{\min_{m=1,\cdots,m}2^{\gamma}h_{1i},2^{\gamma}H_2,\sigma^{\gamma}\right\};$$

$$c=\frac{m}{2}\left(l^2+L_2\right)+\frac{1}{2}\sum_{i=1}^{m}\varepsilon_i^2+\sum_{1=1} m\frac{ml_{1i}}{k_{ai}^2-v_{1i}^2}+\frac{\sigma\theta^2}{r}+1\text{。}$$

Equation (27) can be rewritten as

$$\dot{V}_2 \le -\left(a-\frac{c}{2V_2}\right)V_2-\left(b-\frac{c}{2}V_2^{-\gamma}\right)V_2^{\gamma} \qquad (28)$$

According to Equation (28), make $a-\frac{c}{2V_2}>0$, and $b-\frac{c}{2}V_2^{-\gamma}>0$ by selecting appropriate parameters. Based on Lemma 1, it can be known that $v_j(j=1,\cdots,m)$ and $\tilde{\theta}$ semi - global are asymptotically stable in finite time and will converge in finite time T1to $|v_{1i}|\le k_{ai}\sqrt{1-e^{-2\left(\frac{2c}{b}\right)^{\frac{1}{\gamma}}}}<k_{ai}, \mathrm{v}_2\le 2^{\frac{1}{2}}\left(\frac{2c}{b}\right)^{\frac{1}{2\gamma}}, |\tilde{\theta}|\le(2r)\frac{1}{2}\left(\frac{2c}{b}\right)^{\frac{1}{2\gamma}}$, respectively.

According to Literature , the instruction filtering compensation signal $\xi_r\left(r=1,2\right)$ converges in finite time T2to the domain $|\xi_r|\le max\left[\sqrt{c_0/a_0},\sqrt{2\left(c_0/2b_0\right)}\right]=\vartheta$, where $b_0=\min\left\{\min_{m=1,\cdots,m}2^{1/2}l_{1i},2^{1/2}L_2\right\}, b_0=\min\left\{\min_{m=1,\cdots,m}2^{1/2}l_{1i},2^{1/2}L_2\right\}, c_0=\frac{1}{2}\beta^2$.

According to $v_1=z_1-\xi_1$, $|z_{1i}|\le|v_{1i}|+|\xi_{1i}|\le k_{ai}+\vartheta$ .By the same token, because $z_1=x_1-x_r$, then $|x_1|\le|z_1|+|x_r|\le k_{ai}+\vartheta+Y_0=k_{c1}$. Based on the above analysis, the output state variable x of the mechanical arm system is constrained in the compact set $\Omega_x$, which ensures the finite time output constraint of the mechanical arm.

**V. Simulation Result Analysis**

To verify the effectiveness of the proposed control scheme, the two-degree-of-freedom robotic arm on the vertical plane (Figure 1) is taken as the experimental object. The mechanical arm works in a confined space and there is a wall at x=0.8.

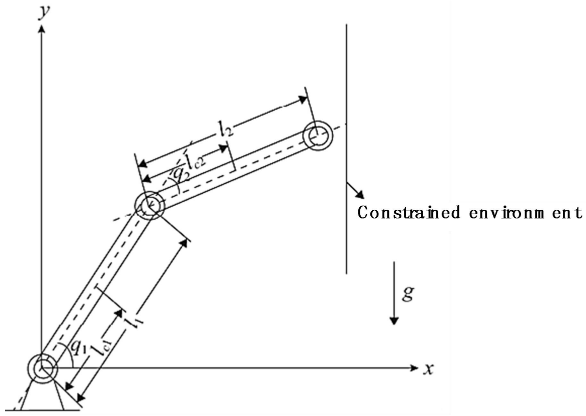


*Fig 1: Mechanical mode of a two-degree-of-freedom mechanical arm*

MATLAB was used for the simulation analysis of the proposed control method. The finite-time output constraint impedance control (FTOCIC) method proposed in this paper was simulated and compared with the fuzzy adaptive instruction filter impedance control method without considering the finite-time control and output state constraints. According to Equation (6), the system model of the rotating joint 2-DOF mechanical arm in the simulation experiment is shown as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = D_x^{-1} \left[ \tau_x - \tau_{fx} - \tau_d - F_e - C_x x_2 - G_x \right] \end{cases}$$

Where $x_1 = \left[ x_{1,1}、 x_{1,2} \right]^T$ ,$x_{1,1}$、 $x_{1,2}$ represent the position of the end of the 2-DOF mechanical arm on the xy axis of the Cartesian coordinate system. The inertia matrix D(q), Coriolis force and centrifugal force matrix, gravity term matrix G(q) and Jacobian matrix J of the 2-DOF mechanical are defined as follows:

$$J = \begin{bmatrix} -\left( l_1 \sin q_1 + l_2 \sin(q_1 + q_2) \right) & -l_2 \sin(q_1 + q_2) \\ l_1 \cos q_1 + l_2 \cos(q_1 + q_2) & l_2 \cos(q_1 + q_2) \end{bmatrix}$$

$$D(q) = \begin{bmatrix} m_1 l_{c1}^2 + m_2 l_{c1}^2 + l_{c2}^2 + 2 l_1 l_{c2} \cos q_2 + I_1 + I_2 & m_2 \left( l_{c2}^2 + 2 l_1 l_{c2} \cos q_2 \right) + I_2 \\ m_2 \left( l_{c2}^2 + 2 l_1 l_{c2} \cos q_2 \right) + I_2 & m_2 l_{c1}^2 + I_2 \end{bmatrix}$$

$$C(q, \dot{q}) = \begin{bmatrix} -m_2 l_1 l_{c2} \dot{q}_2 \sin q_2 & -m_2 l_1 l_{c2} \left( \dot{q}_1 + \dot{q}_2 \right) \sin q_2 \\ m_2 l_1 l_{c2} \dot{q}_{c1} \sin q_2 & 0 \end{bmatrix}$$

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$$G(q)=\begin{bmatrix}(m_1 l_{c2}+m_2 l_{c1}) g\cos q_1+m_2 l_{c2} g(\dot{q}_1+\dot{q}_2)\sin q_2 \\ m_2 l_{c2} g(q_1+q_2)\sin q_2\end{bmatrix}$$

Where $q_i$ represents the i-th joint angle of the mechanical arm; $m_i$ and $l_i$ are the mass and length of the i-th connecting rod of the mechanical arm; $l_{ci}$ is the distance from joint i-1 of the mechanical to the center of mass of the ith connecting rod; I is the moment of inertia of joint i based on the axis passing through the center of mass of the joint (i =1,2).

The parameters of the 2-DOF mechanical arm are shown in Table 1. The initial parameters of the mechanical arm are $x_i=[0.502,0.702]^T, x_2=[0,0]^T$, and the initial values of the filtering compensation signal are $\xi_i=[0,0]^T, \xi_2=[0,0]^T$.

The command trajectory at the end of the 2-DOF mechanical arm is selected as $x_d=[0.7+0.2cos(t)\ ,0.7+0.2sin(t)]^T$. The friction force and interference vector of the mechanical arm are selected as, $\tau_{fx}=[0.1\dot{q}_1,0.1\dot{q}_2]^T$ and $\tau_d=[\ sin(t)\cos(t)\ +sin(t)\ +0.5,2sin(t)\ cos(t)\ +cos(t)\ +1]^T$, respectively.

Then t∈[0,20].

*Table 1 Parameters of the two-degree-of-freedom mechanical arm*

| Parameter | Description | Numerical value |
|---|---|---|
| $m_1$/kg | Mass of connecting rod 1 | 2.00 |
| $m_2$/kg | Mass of connecting rod 2 | 0.85 |
| $l_1$ /m | Length of connecting rod 1 | 1.00 |
| $l_2$/m | Length of connecting rod 2 | 0.80 |
| $I_1$/(kg·m$^2$) | Rotational inertia of connecting rod 1 | 0.500 |
| $I_2$/(kg·m$^2$) | Rotational inertia of connecting rod 2 | 0.136 |

For the fuzzy adaptive instruction filtering impedance control method of two-degree-of- freedom mechanical arm, the control law $k_1=20, k_2=560, r=1, l=1, \sigma=0.1$ is selected.

For the fuzzy adaptive finite time output constraint instruction filter impedance control method of a 2 DOF mechanical arm, the control parameters are $H_1=\text{diag}[0.1,0.1]$, $H_2=2, L_1=\text{diag}\ [0.01,0.01]$, $L_2=2$, $K_2=560$, $ka=[0.005,0.005]^{\mathrm{T}}, r=1, l=1, \sigma=0.1, \gamma=0.6, R_1=20, R_2=0.6$.

The expected impedance parameters of the 2-DOF mechanical arm are $M_d=2I, B_d=\text{diag}\ [20,20], K_d=\text{diag}\ [50,50]$, and the fuzzy logic system selects the fuzzy set as

$$S_j(z) = \exp\left[\left(z_j + s\right)/4\right]$$

Where j=1,2,3,4;s=-3,-2,-1,0,1,2,3.

The simulation results are shown in Figures 2-5. Figure 2 shows the position tracking curve of the mechanical arm end with the proposed control method and contrast control method and the contact force curve applied by the external environment to the mechanical arm end. It can be seen from Figure 2 that the proposed control method can effectively make the end of the mechanical arm track the desired trajectory. Figure 3 shows the comparison diagram of tracking error between the proposed control method and the comparison control method in this paper, as well as the comparison diagram of the change curve of error variable $v_1$.As can be seen from Figure 3, the proposed control method proposed effectively improves the response speed of the system, reduces the position tracking error, and keeps the error variable $v_1$within a given bounded interval. Figure 4 shows the comparison diagram of speed tracking error at the end of the mechanical arm between the proposed control method and the comparison control method, and the comparison diagram of error variable $v_2$ change curve. Figure 5 shows the curves of the real control law $\tau_x$ between the proposed control method and the contrast control method.

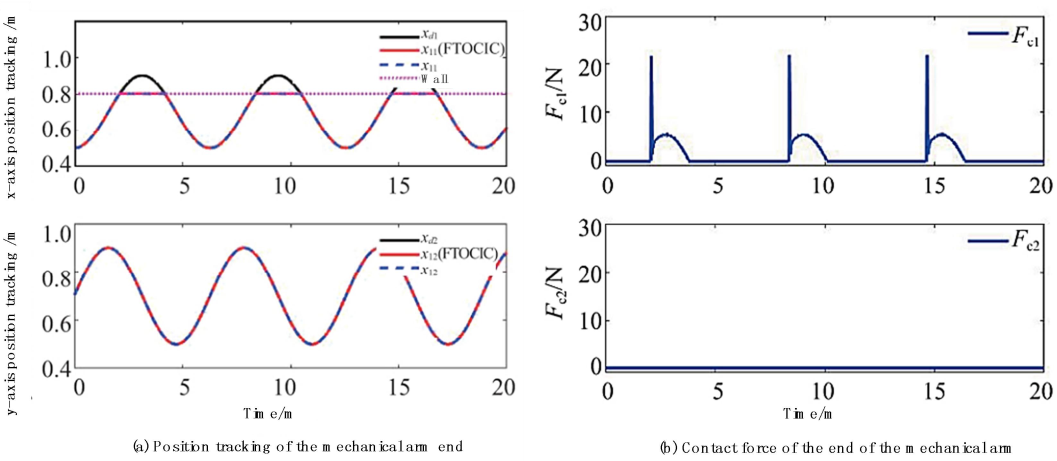


*Fig 2: End position and contact force of the mechanical arm*

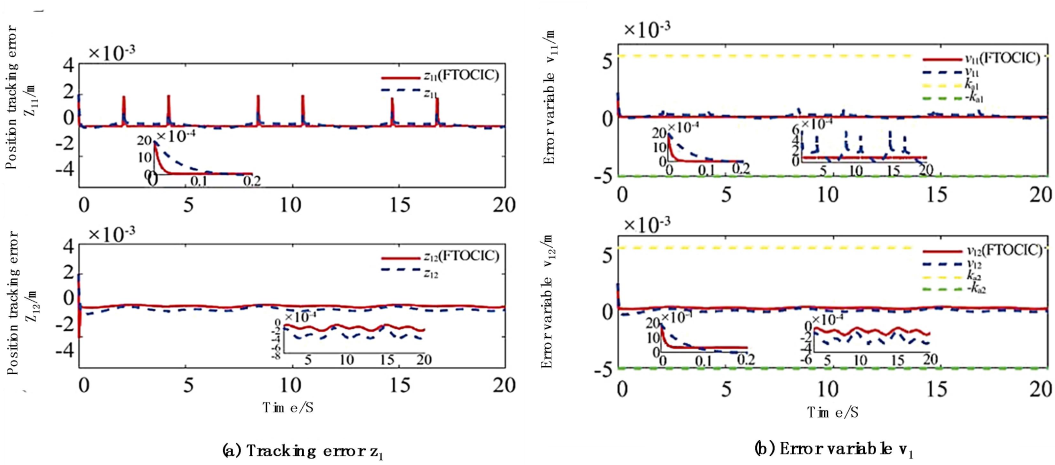


*Fig 3: Tracking error z1 and error variable v1*

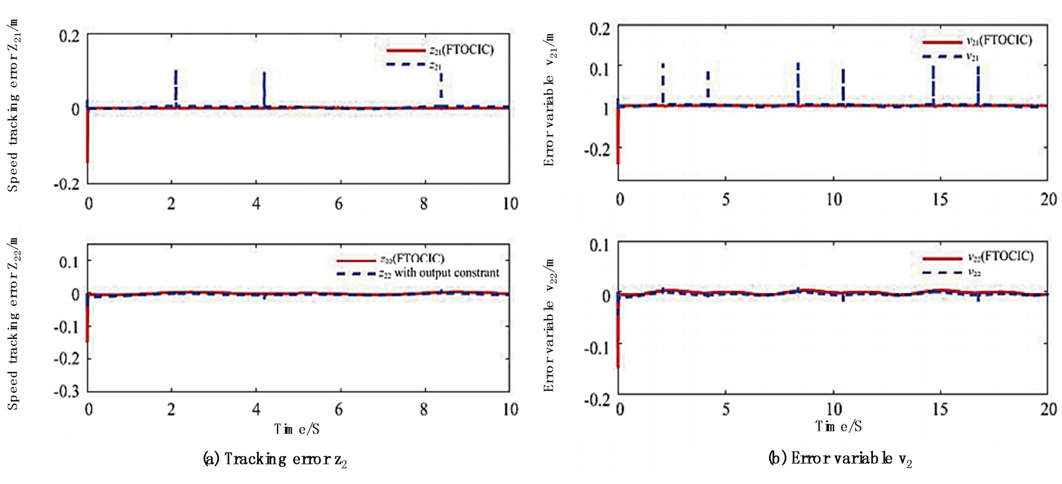


*Fig 4: Tracking error z2 and error variable v2*

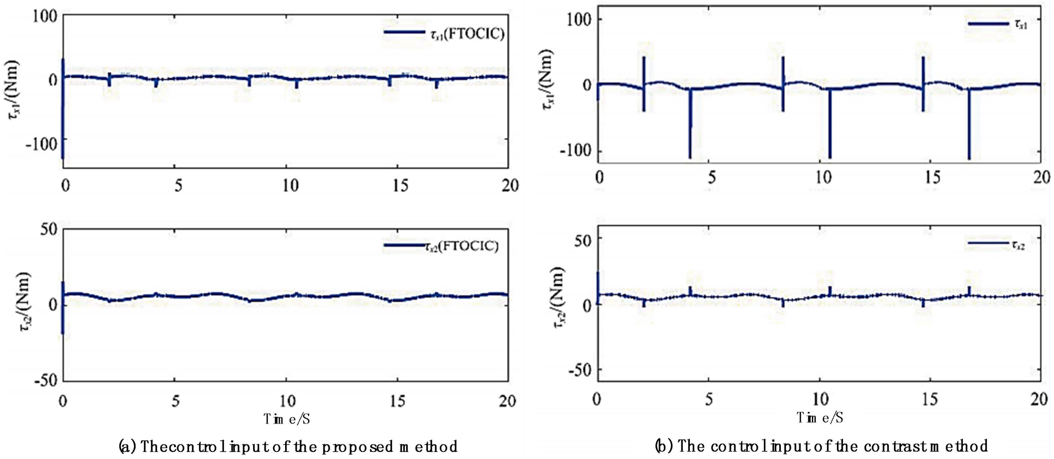


*Fig 5: Control input τx*

## VI. Conclusion

To solve the tracking control problem of the mechanical arm system in a limited space, the proposed output finite time constraint based on instruction filter fuzzy adaptive impedance control method, realized the effective tracking control of the desired trajectory, and the mechanical arm output state was limited in the predefined constraint space, so it had a faster response speed and a smaller tracking error.Compared with the existing control methods, the proposed control method can achieve less response time and better tracking effect, and the output of the mechanical arm can be constrained in a given bounded interval to guarantee the safety of the mechanical arm during operation. The simulation results compared and verified the effectiveness of the proposed control method.

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